

Question 1

Find the distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(1 - (-3))^2 + (-2 - 1)^2} \\
 &= \sqrt{(4)^2 + (3)^2} \\
 d &= \sqrt{16 + 9} \\
 d &= \sqrt{25} \Rightarrow \underline{d = 5}
 \end{aligned}$$

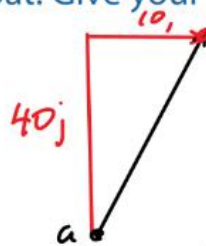
Bronze

**Question 2**

A ship sails at a velocity of 40km/h north. Water flows 10km/h due west.

Calculate the resultant velocity of the boat. Give your answer to 2 decimal places.

$$\begin{aligned}
 \vec{a} &= 10\mathbf{i} + 40\mathbf{j} \\
 &= \begin{pmatrix} 10 \\ 40 \end{pmatrix}
 \end{aligned}$$



$$\begin{aligned}
 |a| &= \sqrt{10^2 + 40^2} \\
 |a| &= \sqrt{100 + 1600} \\
 &= \sqrt{1700} \\
 &= \underline{41.231}
 \end{aligned}$$

Bronze



Question 3

Given the vectors $\underline{a} = 3\mathbf{i} + 4\mathbf{j}$, $\underline{b} = 2\mathbf{i} + 5\mathbf{j}$ and $\underline{c} = 3\mathbf{i} + 3\mathbf{j}$, determine the magnitude of their resultant.

Sum

$$\underline{a} + \underline{b} + \underline{c} = 3\mathbf{i} + 2\mathbf{i} + 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{j} + 3\mathbf{j}$$

$$= 8\mathbf{i} + 12\mathbf{j}$$

Magnitude

$$|\underline{a} + \underline{b} + \underline{c}| = \sqrt{8^2 + 12^2} = \sqrt{64 + 144} = \sqrt{208}$$

$$\begin{array}{r} \sqrt{208} \\ \swarrow \searrow \\ 16 \quad 13 \\ \swarrow \searrow \\ 4 \quad 13 \\ \swarrow \searrow \\ 2 \quad 13 \end{array} = \sqrt{16} \times \sqrt{13} = 4\sqrt{13}$$

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Question 4

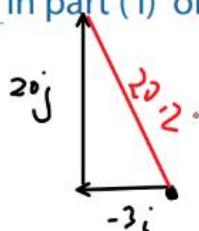
Given vectors $\underline{a} = 4\mathbf{i} + 10\mathbf{j}$, $\underline{b} = 15\mathbf{i} + 10\mathbf{j}$

(i) Find the vector and $\underline{c} = 3\underline{a} - \underline{b}$ and its magnitude correctly to 3 significant figures.

$$\underline{c} = 3(4\mathbf{i} + 10\mathbf{j}) - (15\mathbf{i} + 10\mathbf{j}) \rightarrow \underline{c} = 3\begin{pmatrix} 4 \\ 10 \end{pmatrix} - \begin{pmatrix} 15 \\ 10 \end{pmatrix}$$

$$= 12\mathbf{i} + 30\mathbf{j} - 15\mathbf{i} - 10\mathbf{j} = -3\mathbf{i} + 20\mathbf{j} \quad \underline{\underline{\underline{c} = \begin{pmatrix} -3 \\ 20 \end{pmatrix}}}$$

(ii) Represent vector $\underline{c}$ in part (i) on the x-y plane.



$$|\underline{c}| = \sqrt{(-3)^2 + (20)^2} = \sqrt{9 + 400} = \sqrt{409} = 20$$

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Question 5

Given $\underline{a} = 2\mathbf{i} + 3\mathbf{j}$, $\underline{b} = -6\mathbf{i} - 2\mathbf{j}$ and $\underline{c} = 4\mathbf{i} + 6\mathbf{j}$, find:

i) the vector $\underline{d} = 2\underline{a} - \underline{b} + \frac{1}{2}\underline{c}$

$$\underline{d} = 2(2\mathbf{i} + 3\mathbf{j}) - (-6\mathbf{i} - 2\mathbf{j}) + \frac{1}{2}(4\mathbf{i} + 6\mathbf{j})$$

$$= 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -6 \\ -2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

$$= 4\mathbf{i} + 6\mathbf{j} + 6\mathbf{i} + 2\mathbf{j} + 2\mathbf{i} + 3\mathbf{j}$$

$$\underline{d} = 12\mathbf{i} + 11\mathbf{j} = \begin{pmatrix} 12 \\ 11 \end{pmatrix}$$

(ii) a unit vector in the direction of $\underline{d}$

$$\frac{\underline{d}}{|\underline{d}|} = \frac{12\mathbf{i} + 11\mathbf{j}}{\sqrt{12^2 + 11^2}} = \frac{12\mathbf{i} + 11\mathbf{j}}{\sqrt{144 + 121}} = \frac{12\mathbf{i} + 11\mathbf{j}}{\sqrt{265}}$$

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Question 6

If $\underline{a} = 3\mathbf{i} + 2\mathbf{j}$, $\underline{b} = 5\mathbf{i} - 11\mathbf{j}$ and $\underline{c} = 3\mathbf{i} - 5\mathbf{j}$, find the value of x and y such that $x\underline{a} + y\underline{c} = \underline{b}$.

$$x\begin{pmatrix} 3 \\ 2 \end{pmatrix} + y\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ -11 \end{pmatrix}$$

$$\textcircled{1} 3x + 3y = 15$$

$$\textcircled{2} 2x - 5y = -11$$

sol ①

$$3x + (3 \times 3) = 15$$

$$3x + 9 = 15$$

$$3x = 6$$

$$x = 2$$

$$\text{NEW } \textcircled{1} 6x + 6y = 30$$

$$+ \textcircled{2} 6x - 15y = -33$$

$$\hline 21y = 63 \therefore y = \frac{63}{21} = 3$$

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