

c) Simplify $\frac{2}{(1+\sqrt{5})} \times \frac{(1-\sqrt{5})}{(1-\sqrt{5})} = \frac{2-2\sqrt{5}}{1-\sqrt{5}+\sqrt{5}-5} = \frac{2-2\sqrt{5}}{-4}$

$$= \frac{2}{-4} - \frac{2\sqrt{5}}{-4} = -\frac{1}{2} + \frac{1}{2}\sqrt{5} = \frac{\sqrt{5}-1}{2}$$

d) Simplify $\frac{2}{(3\sqrt{2}-1)} \times \frac{(3\sqrt{2}+1)}{(3\sqrt{2}+1)} = \frac{6\sqrt{2}+2}{18+3\sqrt{2}-3\sqrt{2}-1} = \frac{6\sqrt{2}+2}{17}$

Work out the value of x

a) $2^x = \frac{1}{8} = 2^{-3} = \frac{1}{8}$

Hence work out the value of $a + b + c$

b) $2^a = \frac{1}{8}, 2^b = 8\sqrt{2}, 2^c = \frac{1}{2\sqrt{2}} = -1$

$$a = -3, \quad 2^3 \times 2^{\frac{1}{2}} = 2^{3.5} \quad \frac{1}{2^1 \times 2^{\frac{1}{2}}} = \frac{1}{2^{1.5}} \quad c = -1.5$$

Question 3

Simplify

Core 3 - Bronze

Topic: Indices and Logs

$$\log_a(b) = x \Rightarrow a^x = b$$

a) $\log_3 3 + \log_3 9 =$ $\log_3(x) + \log_3(y) = \log_3(xy) = \log_3(27) = x$
 $3^x = 27 \therefore \underline{x = 3}$

b) $\log_2 12 - \log_2 3 =$ $\log_2\left(\frac{12}{3}\right) = \log_2 4$

c) $\log 3^x =$ $x \log 3$

Work out the value of x

d) $\log_2 x = 3$

$2^3 = 8 \therefore x = 8$

Bronze



Question 4

Work out the value of x

Core 4 - Bronze

Topic: Indices and Logs

a) $3^3 \times 9 = 3^x$

$3^3 \times 3^2 = 3^x$ $3+2 = x \therefore \underline{x = 5}$

b) $16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$

$(2^4)^{\frac{1}{5}} \times (2)^x = (2^{\frac{3}{4}})^{\frac{3}{4}}$, $2^{\frac{4}{5}} \times 2^x = 2^{\frac{9}{4}}$
 $\frac{4}{5} + x = \frac{9}{4}$

c) $4^{(2x-4)} = 8^{(x+2)}$

$(2^2)^{(2x-4)} = (2^3)^{(x+2)}$, $2^{4x-8} = 2^{3x+6}$
 $4x-8 = 3x+6$
 $x = 14$

$x = \frac{9}{4} - \frac{4}{5}, \frac{45-16}{20} = \frac{29}{20} = 1\frac{9}{20}$

Question 5

Show that $\frac{2 - 4\sqrt{2}}{2 + 2\sqrt{2}}$ can be simplified to $3\sqrt{2} - 5$

$$\begin{aligned} \frac{(2-4\sqrt{2})}{(2+2\sqrt{2})} \times \frac{(2-2\sqrt{2})}{(2-2\sqrt{2})} &= \frac{4 - 4\sqrt{2} - 8\sqrt{2} + [8 \times 2]}{4 - 2\sqrt{2} + 2\sqrt{2} - [4 \times 2]} \\ &= \frac{4 - 12\sqrt{2} + 16}{4 - 8} = \frac{20 - 12\sqrt{2}}{-4} \\ &= \frac{20}{-4} + 3\sqrt{2} \\ &= 3\sqrt{2} - 5 \end{aligned}$$

Question 6

Work out the value of n

a) $n \log_3 81 = \log_2 8$

$$\begin{aligned} n \log_3 (81) &= \log_2 (8) \\ n \log_3 (3^4) &= \log_2 (2^3) \\ 4n \log_3 (3) &= 3 \log_2 (2) \\ 4n &= 3 \\ n &= \frac{3}{4} \end{aligned}$$

Silver



Question 7

Work out the value of y

a) $\log_3 (4y - 2) - 2 = \log_3 (y + 2)$

$$\log_3 (4y - 2) - \log_3 (y + 2) = 2$$

$$\log_3 \left(\frac{4y - 2}{y + 2} \right) = 2$$

$$3^2 = \frac{4y - 2}{y + 2}$$

$$\frac{9}{1} = \frac{4y - 2}{y + 2}$$

$$9(y + 2) = 4y - 2$$

$$9y + 18 = 4y - 2$$

$$9y - 4y = -2 - 18$$

$$5y = -20$$

$$\underline{y = -4}$$

Silver



Question 8

Work out the value of

a) $8^{\frac{1}{4}} \times (\sqrt{2})^{-4} \times 2^{\frac{3}{2}}$

$$= (2^3)^{\frac{1}{4}} \times (2^{\frac{1}{2}})^{-4} \times 2^{\frac{3}{2}}$$

$$= 2^{\frac{3}{4}} \times 2^{-2} \times 2^{\frac{3}{2}}$$

$$= 2^{\frac{3}{4}} \times 2^{-2} \times 2^{\frac{3}{2}}$$

$$= \frac{3}{4} + -\frac{2}{1} + \frac{3}{2} = \frac{3}{4} - \frac{8}{4} + \frac{6}{4} = \underline{\underline{\frac{1}{4}}}$$

Silver



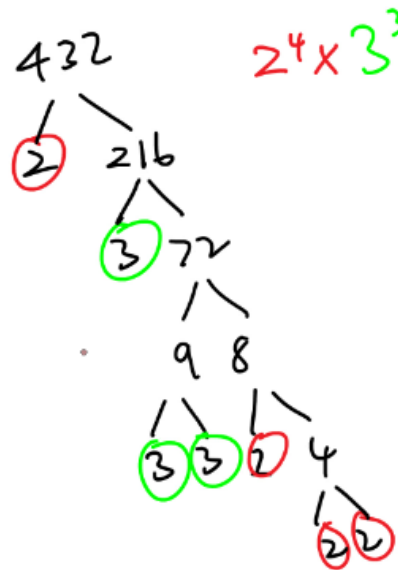
Question 9

Work out the value of x and y

a) $\frac{2^{x-2}}{3^{3y}} = 432$

$$2^{x-2} \times 3^{-3y} = 2^4 \times 3^3$$

$$\begin{array}{l} 2: x-2 = 4 \quad \therefore x = 6 \\ 3: -3y = 3 \quad \therefore y = -1 \end{array}$$



Silver

Question 10

Given that $\log 3 = 0.4771$ and $\log 2 = 0.301$, find the value of $\log(3^{3/8})$, without using mathematical tables.

$$\begin{aligned} \log(3^{3/8}) &= \log\left(\frac{27}{8}\right) = \log 27 - \log 8 \\ &= \log 3^3 - \log 2^3 \\ &= 3\log 3 - 3\log 2 \\ &= 3(0.4771) - 3(0.301) \\ &= 0.5283 \end{aligned}$$

Gold

Question 11

Express the expression

$$a) (2)^{3y-1} + 3 \times (2)^{y+2}$$

in terms of P given that $P = (2)^y$

$$\begin{aligned} & (2)^{3y} \times \underline{2^{-1}} + 3 \times (2)^y \times \underline{2^2} \\ & \frac{1}{2} (2)^{3y} + 12 (2)^y \\ & = \underline{\underline{\frac{1}{2} P^3 + 12 P}} \end{aligned}$$

Gold



Question 12

a) Work out the value of x

$$\sqrt{4^{3x-3}} + 8 = 72$$

$$\sqrt{4^{3x-3}} = 72 - 8$$

$$\sqrt{4^{3x-3}} = 64$$

$$4^{3x-3} = (4^3)^2$$

$$\cancel{4}^{3x-3} = \cancel{4}^6$$

$$\begin{aligned} 3x - 3 &= 6 \\ 3x &= 9 \\ \underline{x &= 3} \end{aligned}$$

Gold



Question 12

a) Work out the value of x

$$\sqrt{4^{3x-3}} + 8 = 72$$

$$\sqrt{4^{3x-3}} = 72 - 8$$

$$\sqrt{4^{3x-3}} = 64$$

$$4^{3x-3} = (4^3)^2$$

$$4^{3x-3} = 4^6$$

$$3x - 3 = 6$$

$$3x = 9$$

$$x = 3$$

Gold



Question 13

Evaluate $\log_{10}(13500)$ given that $\log_{10}2 = 0.301$, $\log_{10}3 = 0.4771$

$$\log_{10}5 = 0.699$$

$$\begin{array}{r} 13500 \\ \diagup \end{array}$$

$$13500 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$\log_{10}(2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5)$$

$$\log_{10}(2^2 \times 3^3 \times 5^3)$$

$$\log_{10}(xyz) = \log_{10}(x) + \log_{10}(y) + \log_{10}(z)$$

$$\begin{aligned} & \log_{10}2^2 + \log_{10}3^3 + \log_{10}5^3 \\ & 2\log_{10}2 + 3\log_{10}3 + 3\log_{10}5 \\ & = 2(0.301) + 3(0.4771) + 3(0.699) \\ & = 0.602 + 1.4313 + 2.097 \\ & = 4.1303 \end{aligned}$$

Platinum

