ANSWERS

Sequences and Series

Maths Memory Tanzania
Question 1

Core 1- Bronze   Topic: Sequences and Series

a) Determine which series are arithmetic and geometric?

1) 5,10,15,20   (A)

2) 3,6,12,24   (G)

3) 12,6,3,1.5   (G)

4) -2,3,8,13   (A)

5) What is the fundamental difference between an arithmetc series and a geometric series?

$+,-$ $\times$

Bronze

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Question 2

Core 2- Bronze   Topic: Sequences and Series

a) The first number of an **arithmetic** sequence is 4. The difference is 6.
State the 50th number in the sequence (using the formula).

$$A_n = A_1 + (n-1)d \qquad \Rightarrow \qquad A_{50} = 4 + (n-1)6$$
$$= 4 + 6(n-1)$$
$$= 4 + 6n - 6$$
$$= 4 - 6 + 6n$$
$$= 6n - 2$$

b) Find the sum of the first 15 numbers (using the formula).

$$S_n = \frac{n}{2}\left[2A_1 + (n-1)d\right]$$
$$= \frac{15}{2} \times \left[(2 \times 4) + ((15-1)6)\right]$$
$$= 7.5 \left[8 + 84\right] \qquad = 7.5 \times 92$$

$$\begin{array}{r} 92 \\ \times 75 \ (14) \\ \hline 46\ 0 \\ 6\ 44\ 0 \\ \hline 6190.0 \end{array} = 690$$

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Question 3

a) The first number of an geometric sequence is 3. The ratio is 2.
State the 6th number in the sequence (using the formula).

$$G_N = G_1 (r)^{N-1}$$
$$= 3 \times (2)^{6-1}$$

$$= 3(2)^5$$
$$= 3 \times 32$$
$$= 96$$

b) Find the sum of the first 6 numbers (using the formula).

$$r > 1$$

$$G_N = G_1 \left[\frac{r^n - 1}{r - 1} \right]$$
$$= 3 \left[\frac{2^6 - 1}{2 - 1} \right]$$

$$3 \times \left[\frac{64 - 1}{2 - 1} \right]$$
$$= 3 \times 6^?$$

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Core 4 - Bronze   **Topic:** Sequences + Series

Question 4

Coby is looking at investing £3,000 in either of these two bank accounts.

Cash account
Compound Interest
3.5% per annum
for 4 years

Shares account
Compound Interest
4% for year 1
5% for year 2
5% for year 3

Which bank will give Coby the most interest at the end of their terms?

$$3,000 \times (1.035)^4 = £3,442.57$$

$$3000 \times (1.04) \times (1.05)^2 = £3,439.80$$

Cash ✓

Calculator

Question 5

Henry invests £4,500 at a compound interest rate of 5% per annum.
At the end of n complete years, the investment has grown to £5743.27. Find the value of n.

Try $n=4$

$4,500 \times (1.05)^4 \neq £5,469.78$ (Low)

Try $n=6$ $4,500 \times (1.05)^6 = £6,030.43$ (High)

Try $n=5$ $4,500 \times (1.05)^5 = £5,743.27$

Calculator

Question 6

Natalie invests £5,500 for 5 years. The investment gets compound interest of x % per annum.
At the end of 5 years the investment is worth £7029.35

Calculate the value of x (Give your answer to 2 decimal places).

$5,500 \times x^5 = 7,029.25$

$x^5 = 1.2781$ (4dp)

$x = (1.2781)^{\frac{1}{5}}$

$x = 1.05029306$

$x = 5.03$

Calculator

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Question 7

A_1 d $7, 13, 19, 25 \ldots$

a) Find the first term and the common difference of an arithmetic progression whose 4th term is 25 and the 7th term is 43.

$$A_n = A_1 + (n-1)d$$
$$A_4 = A_1 + (4-1)d$$
$$\boxed{① \quad 25 = A_1 + 3d}$$

$$A_7 = A_1 + (7-1)d$$
$$\boxed{②\,43 = A_1 + 6d}$$

$$① \quad A_1 + 3d = 25$$
$$② \quad A_1 + 6d = 43$$
$$3d = 18$$
$$d = 6$$

sub ① $A_1 + (3 \times 6) = 25$
$A_1 + 18 = 25$
$A_1 = 7$

Silver

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Question 8

a) The third and fifth terms of Geometric Progression (G.P.) are 32 and 128 respectively. Find the common ratio and the first term of the G.P.

$$G_n = G_1 (r)^{n-1}$$
$$① \quad 32 = G_1 (r)^{3-1}$$
$$32 = G_1 (r)^2$$

$$② \quad 128 = G_1 (r)^{5-1}$$
$$128 = G_1 (r)^4$$

$$① \quad 128 = G_1 r^4$$
$$② \quad 32 = G_1 r^2$$
$$4 = r^2$$
$$r = 2$$

Sub ②
$32 = G_1 (2)^2$
$32 = 4 G_1 \quad \therefore G_1 = 8$

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Question 9

a) How many terms of the series 7,11,15 are needed to make a sum of 168.

$+4 +4$

$$S_N = \frac{N}{2}[2A_1 + (N-1)d]$$

$$\frac{168}{1} = \frac{N}{2}[14 + 4[N-1]]$$

$$\frac{168}{1} = N \frac{[14 + 4N - 4]}{2}$$

$$336 = N[10 + 4N]$$

$$336 = 4N^2 + 10N \quad \Rightarrow 0$$

$$\boxed{4N^2 + 10N - 336 = 0}$$

$$\cancel{2}[2N^2 + 5N - 168] = 0$$

$$2N^2 \underbrace{-16N + 21N}_{} - 168$$

$$2N[N-8] + 21(N-8)$$

$$(2N + 21)(N-8) = 0$$

$$N = -\frac{21}{2}, \quad \boxed{N = 8}$$

$$21 \times \frac{-16}{2} = -336$$
$$\frac{21}{2} + \frac{-16}{2} = 5$$

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Question 10

① ② ③
$$4, 1, \frac{1}{4}$$

a) If the sum of n terms of a geometric progression with first term 4 and common ratio 1/4 is 21/4, find the number of terms in the sequence.

$$\frac{1}{4} < 1$$

$$S_N = G_1\left[\frac{1 - r^n}{1 - r}\right]$$

$$\frac{21}{4} = 4\left[\frac{1 - (\frac{1}{4})^n}{1 - \frac{1}{4}}\right]$$

$$\frac{21}{4} = \frac{4(1 - (\frac{1}{4})^n)}{\frac{3}{4}}$$

$$\frac{63}{4} = 16\left(1 - (\frac{1}{4})^n\right)$$

$$\frac{63}{64} = 1 - (\frac{1}{4})^n$$

$$(\frac{1}{4})^n = 1 - \frac{63}{64}$$

$$(\frac{1}{4})^n = \frac{64}{64} - \frac{63}{64}$$
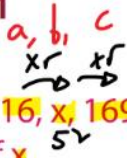
$$(\frac{1}{4})^n = \frac{1}{64}$$

$$n = 3$$

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Question 11

a, b, c

$xr \quad xr^2$

a) Given that 16, x, 169 are consecutive terms of a geometric progression. Find

i) the value of x

52

(ii) the geometric mean

$$\pm\sqrt{ac}$$

$$\pm\sqrt{16 \times 169}$$

$$\pm\sqrt{16} \times \sqrt{169}$$

$$\pm 4 \times 13$$

$$= \pm 52$$

$$\frac{169}{x} = r$$

$$\frac{x}{16} = r$$

$$\frac{x}{16} \diagup\diagdown = \frac{169}{x}$$

$$\Rightarrow x^2 = 169 \times 16$$

$$x = \sqrt{169 \times 16}$$

$$x = \sqrt{169} + \sqrt{16}$$

$$= 13 \times 4$$

$$= 52$$

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Question 12

$$-3, \; \underset{1}{\cancel{-3}}\; \underset{2}{\overset{2}{\bigcirc{4}}}, \underset{3}{11}, \underset{4}{18}, \underset{5}{\bigcirc{25}} \underset{6}{32}, \underset{7}{39}, \underset{8}{46}, \underset{9}{53}, \underset{10}{60}$$

a) The 5th term of an arithmetic progression is 21 greater than the 2nd term. The 10th term is 15 times the 2nd term. Find the common difference and the first term of the arithmetic progression.

$$a_5 = a + 4d$$
$$a_2 = a + d$$
$$\cancel{a} + 4d = \cancel{a} + d + 21$$
$$4d - d = 21$$
$$3d = 21$$
$$\underline{\underline{d = 7}}$$

$$a_{10} = a + 9d$$
$$15(a + d) = a + 9d$$
$$15a + 15d = a + 9d$$
$$14a = -6d$$
$$14a = -42$$
$$a = \frac{42}{14} \Rightarrow \underline{\underline{a = -3}}$$

Platinum